

SOLUTIONS

MAP102 Placement Exam Prep - Algebra

This set of questions is directly related to the material covered in this course. Most of it is straightforward while some of the questions will increase your knowledge base of the topics.

I recommend you complete all the lectures and corresponding homeworks before attempting these.

Remember you will be expected to solve these without a calculator!

1. If $x = 5$ then $3x =$ $3 \cdot 5 = \boxed{15}$

2. If $x = -1$ then $3x + 9 =$ $* \text{fixed}$

$$\begin{aligned} &= 3(-1) + 9 \\ &= -3 + 9 \\ &= \boxed{6} \end{aligned}$$

3. If $3x + 4 = 19$ then $x =$

$$\begin{aligned} 3x + 4 &= 19 \\ -4 &\quad -4 \\ \hline 3x &= \frac{15}{3} \\ x &= \boxed{5} \end{aligned}$$

Exercises on Solving Quadratic Equations

Factorising

4. If $2x^2 + 13 = 63$ then $x =$

$-5, 5$

$$2x^2 + 13 - 63 = 63 - 63$$

$$2x^2 - 50 = 0$$

$$2(x^2 - 25) = 0$$

$$2(x+5)(x-5) = 0$$

$$\cancel{2 \neq 0} \quad x+5=0 \quad x-5=0$$

* Invalid Solution $x = -5$ $x = 5$

5. If $4x - 5 = 3 + 2x$ then $x =$

$$4x - 2x = 3 + 5$$

$$2x = 8$$

$$x = 4$$

6. Fully simplify $3x + 2(x-1) - 3(y-2)$.

$$\begin{aligned}
 &= \underline{3x} + \underline{2x} - \underline{2} - \underline{3y} + \underline{6} && \text{combine like terms} \\
 &= \boxed{\underline{5x} + \underline{4} - \underline{3y}}
 \end{aligned}$$

7. If $a \neq 0, b \neq 0$ and $ax + b = c$ then $x =$

$$\frac{ax}{a} = \frac{c-b}{a}$$

8. The sum of three consecutive integers is 156. What is the largest of these integers?

let $x = 1^{\text{st}}$ integer
 let $x+1 = 2^{\text{nd}}$ integer
 let $x+2 = 3^{\text{rd}}$ integer

↑ largest #

$$\rightarrow \underline{1^{\text{st}} \text{ int.}} + \underline{2^{\text{nd}} \text{ int.}} + \underline{3^{\text{rd}} \text{ int.}} = 156$$

$$\underline{x} + \underline{x+1} + \underline{x+2} = 156 \quad \text{combine like terms}$$

$$3x+3 = 156$$

$$3x = 153$$

$$\frac{x}{x+2} = \frac{51}{53} = \frac{1^{\text{st}} \text{ int.}}{3^{\text{rd}} \text{ int.}}$$

$$3 \sqrt{153}$$

9. A boy buys 12 candies, all of which are either gumballs or jelly beans. Gumballs cost 3 cents each while jelly beans cost 4 cents each. If the boy paid 41 cents in all, how many gumballs did he buy?

let $x = \#$ of gumballs
 let $y = \#$ of jelly beans

$3x =$ cost of gumballs
 (in cents)

$4x =$ cost of jelly bean
 (in cents)

linear sys. of eqns.

$$\left\{ \begin{array}{l} x+y=12 \\ 3x+4y=41 \end{array} \right. \begin{array}{l} \text{mult by -4} \\ \leftarrow \end{array}$$

$$\Leftrightarrow \left\{ \begin{array}{l} -4x-4y=-48 \\ 3x+4y=41 \end{array} \right. \begin{array}{l} \text{add eqns} \\ \end{array}$$

$$\Leftrightarrow \left\{ \begin{array}{l} -x = -7 \\ 3x+4y=41 \end{array} \right. \begin{array}{l} * \text{ mult by -1} \\ \end{array} \quad * \text{fixed}$$

$$\Leftrightarrow \left\{ \begin{array}{l} x = 7 \\ 3x+4y=41 \end{array} \right. \begin{array}{l} \# \text{ gumballs} \\ \end{array}$$

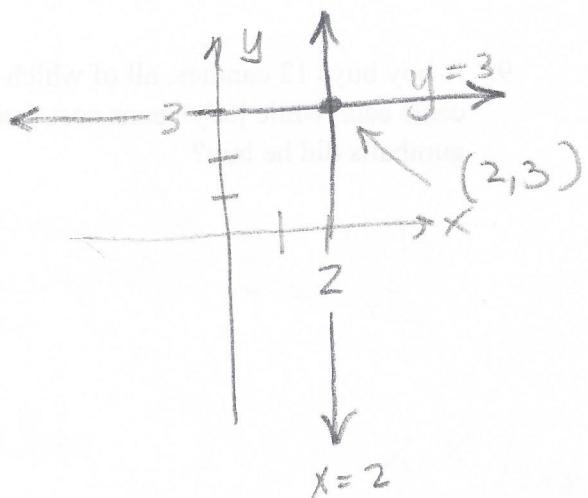
re-write as a fraction

10. If $x \neq 0$ and $y \neq 0$ then $(x^2y^4)/(x^3y^2) =$

$$\begin{aligned} & \frac{x^2y^4}{x^3y^2} \\ &= \frac{\cancel{x^2}\cancel{y^2}\cancel{y^2}}{\cancel{x^2}\times\cancel{y^2}} = \left(\frac{y^2}{x}\right) \end{aligned}$$

11. The straight lines $x=2$ and $y=3$...

- "intersect"
- a. cross at the point $(2,3)$
 - b. cross at the point $(3,2)$
 - c. cross at the point $(5,5)$
 - d. are the same
 - e. do not cross



12. The straight lines $x-y-1=0$ and $x+y-3=0$ cross at the point (\quad, \quad) .

make linear sys of eqns

$$\begin{cases} x-y-1=0 \\ x+y-3=0 \end{cases} \quad \text{add eqn's}$$

$$\Leftrightarrow \begin{cases} 2x-4=0 \\ x+y-3=0 \end{cases} \quad \text{solve for } x$$

$$\Leftrightarrow \begin{cases} x=2 \\ x+y-3=0 \end{cases} \quad \text{plug in } x$$

$$\Leftrightarrow \begin{cases} x=2 \\ 2+y-3=0 \end{cases}$$

$$\begin{cases} x=2 \\ y=1 \end{cases} \Rightarrow (2,1)$$

13. What is the equation of the line which passes through the origin and is parallel to the line

$$y = 3x + 4?$$

$$\downarrow \\ m=3$$

$$(0, 0) \\ x, y_1$$

\downarrow
same slope

$$y - y_1 = m(x - x_1) \\ y - 0 = 3(x - 0) \\ \boxed{y = 3x}$$

$$14. \frac{6n+3n^2}{3n} = \frac{6n}{3n} + \frac{3n^2}{3n}$$

write as 2 separate fractions

$$\frac{2 \cdot 3 \cdot n}{3 \cdot n} + \frac{3 \cdot n \cdot n}{3 \cdot n}$$

$$\boxed{2 + n}$$

Factor

15. Solve for x : $2x^2 + 5x - 3$

factors of 6 must multiply to give 6
subtract to give 5

$$AC = 2 \cdot 3 = 6$$
$$\begin{array}{c} (6 \cdot 1) \\ 2 \cdot 3 \end{array}$$
$$= 2x^2 + \underline{\underline{6x}} - \underline{\underline{1x}} - 3 \quad \text{factor by grouping}$$
$$= 2x^2 + 6x - x - 3 \quad \text{factor pairs}$$
$$= 2x \underline{(x+3)} - 1 \underline{(x+3)} \quad \text{common factor of } x+3$$
$$= \boxed{(x+3)(2x-1)}$$

16. The y -value of the solution of the system $\begin{cases} 2x - y = 9 \\ 3x + 2y = 10 \end{cases}$ is

$$\Leftrightarrow \begin{cases} 4x - 2y = 18 & \text{add eqns} \\ 3x + 2y = 10 \end{cases}$$

$$\Leftrightarrow \begin{cases} 4x - 2y = 18 \\ 7x = 28 \end{cases} \quad \text{divide by 7}$$

$$\Leftrightarrow \begin{cases} 4x - 2y = 18 \\ x = 4 \end{cases} \quad \text{plug in}$$

$$\Leftrightarrow \begin{cases} 4(4) - 2y = 18 & \text{solve for } y \\ x = 4 \end{cases}$$

$$\begin{cases} y = -1 \\ x = 4 \end{cases}$$

$(4, -1)$

17. Which of the following are factors of $x^4 - 16$? \leftarrow difference of two squares

$$= (x^2 + 4)(x^2 - 4)$$

$$= (x^2 + 4)(x - 2)(x + 2)$$

- (I) $x - 2$ (II) $x + 2$ (III) $x^2 + 4$

- a. I only
- b. III only
- c. II only
- d. I and II only
- e. I, II and III

18. The solutions to $x^3 + 2x^2 - 3x = 0$ are GCF is x

$$x(x^2 + 2x - 3) = 0$$

$$x(x+3)(x-1) = 0$$

* fixed

$x = 0$	$x = -3$	$x = 1$
---------	----------	---------

19. If $x > 0$ then factor and simplify $\frac{x^2 - x - 6}{x^2 + 5x + 6}$.

$$= \frac{x-3}{x+3}$$

20. The length of a rectangle is 3 inches more than twice its width. If the perimeter is 30 inches then the width of the rectangle is ...

$$\begin{aligned} \text{let width} &= w \\ \text{let length} &= 3 + 2w \end{aligned}$$

$$\begin{aligned} P &= 2w + 2l \\ 30 &= 2w + 2(3 + 2w) \end{aligned}$$

distribute

$$30 = 2w + 6 + 4w$$

$$\begin{aligned} 24 &= 6w \\ 4 &= w \end{aligned}$$

21. Mary has two different kinds of clover in her garden. Some have three leaves and others have four leaves. Mary has a total of 218 clovers and these clovers have a total of 660 leaves. How many four-leaf clovers does Mary have?

Let $x = \#$ three-leaved clovers
 $y = \#$ four-leaved clovers

$$\begin{cases} x+y = 218 \\ 3x+4y = 660 \end{cases}$$

mult by -3

$$\Leftrightarrow \begin{cases} -3x - 3y = -654 \\ 3x + 4y = 660 \end{cases}$$

add $\frac{218}{\times 3} \quad \frac{-654}{654}$

$$\Leftrightarrow \begin{cases} y = 6 \\ 3x + 4(6) = 660 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = 6 \quad \boxed{4\text{-leaf clovers}} \\ 3x + 4(6) = 660 \end{cases}$$